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1. Examine the continuity of the function f (x) = $x^3 + 2x^2 - 1$ at x = 1

Solution:

We know that, y = f(x) will be continuous at x = a if,

We know that $y = f(x)$ will be continuous at $x = a$ if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a} f(x) = \lim_{x \to a^+} f(x)$$

Given: $f(x) = x^3 + 2x^2 - 1$

$$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} (1+h)^3 + 2(1+h)^2 - 1 = 1 + 2 - 1 = 2$$

$$\lim_{x \to 1} f(x) = (1)^3 + 2(1)^2 - 1$$

$$= 1 + 2 - 1 = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{\to} (1+h)^3 + 2(1+h)^2 - 1$$

$$= 1 + 2 - 1 = 2$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} f(x) = \lim_{x \to 1^+} f(x) = 2.$$

Hence, $f(x)$ is continuous at $x = 1$.

Thus, f(x) is continuous at x = 1.

Find which of the functions in Exercises 2 to 10 is continuous or discontinuous at the indicated points:

2.

$$f(x) = \begin{cases} 3x + 5, \text{ if } x \geq 2 \\ x^2, \text{ if } x < 2 \end{cases}$$

at x = 2

Solution:

Checking the continuity of the given function, we have

$$\lim_{x \to 2^{+}} f(x) = 3x + 5$$
$$= \lim_{h \to 0} 3(2 + h) + 5 = 11$$

$$\lim_{x \to 2} f(x) = 3x + 5 = 3(2) + 5 = 11$$

$$\lim_{x \to 2^{-}} f(x) = x^2 = \lim_{h \to 0} (2 - h)^2$$
$$= \lim_{h \to 0} (2)^2 + h^2 - 4h = (2)^2 = 4$$

Now, since $\lim_{x \to 2^{-}} f(x) = \lim_{x \to 2} f(x) \neq \lim_{x \to 2} f(x)$

Thus, f(x) is discontinuous at x = 2.

3.

$$f(x) = \begin{cases} \dfrac{1 - \cos 2x}{x^2}, & if\ x \neq 0 \\ 5, & if\ x = 0 \end{cases}$$ **at x = 0**

Solution:

Checking the right hand and left hand limits of the given function, we have

$$\lim_{x \to 0^-} f(x) = \frac{1 - \cos 2x}{x^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos 2(0 - h)}{(0 - h)^2} = \lim_{h \to 0} \frac{1 - \cos(-2h)}{h^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos 2h}{h^2}$$

$$= \lim_{h \to 0} \frac{2 \sin^2 h}{h^2} \qquad \left[\because 1 - \cos\theta = 2\sin^2\frac{\theta}{2}\right]$$

$$= \lim_{h \to 0} \frac{2 \sin h}{h} . \frac{\sin h}{h} = 2.1.1 = 2 \qquad \left[\lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$\lim_{x \to 0^+} f(x) = \frac{1 - \cos 2x}{x^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos 2(0 + h)}{(0 + h)^2} = \lim_{h \to 0} \frac{1 - \cos 2h}{h^2}$$

$$= \lim_{h \to 0} \frac{2 \sin^2 h}{h^2} = \frac{2 \sin h}{h} . \frac{\sin h}{h} = 2.1.1 = 2$$

$$\lim_{x \to 0} f(x) = 5$$

As $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) \neq \lim_{x \to 0} f(x)$

Therefore, the given function f(x) is discontinuous at x = 0.

4.

$$f(x) = \begin{cases} \dfrac{2x^2 - 3x - 2}{x - 2}, & if\ x \neq 2 \\ 5, & if\ x = 2 \end{cases}$$ **at x = 2**

Solution:

The given fucntion at x $\neq$ 0 can be rewritten as,

$$f(x) = \frac{2x^2 - 3x - 2}{x - 2}$$

$$= \frac{2x^2 - 4x + x - 2}{x - 2} = \frac{2x(x-2) + 1(x-2)}{x-2}$$

$$= \frac{(2x+1)(x-2)}{x-2} = 2x + 1$$

Now,

$$\lim_{x \to 2^-} f(x) = 2x + 1$$

$$= \lim_{h \to 0} 2(2 - h) + 1 = 4 + 1 = 5$$

$$\lim_{x \to 2^+} f(x) = 2x + 1$$

$$= \lim_{h \to 0} 2(2 + h) + 1 = 4 + 1 = 5$$

$$\lim_{x \to 2} f(x) = 5$$

As $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2} f(x) = 5$

The given fucntion at x $\neq$ 0 can be rewritten as,

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Now,

$$\lim_{x \to 2^-} f(x) = 2x + 1$$

$$= \lim_{h \to 0} 2(2 - h) + 1 = 4 + 1 = 5$$

$$\lim_{x \to 2^+} f(x) = 2x + 1$$

$$= \lim_{h \to 0} 2(2 + h) + 1 = 4 + 1 = 5$$

$$\lim_{x \to 2} f(x) = 5$$

As $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2} f(x) = 5$

Thus, f(x) is continuous at x = 2.

5.

$$f(x) = \begin{cases} \frac{|x-4|}{2(x-4)}, & if\ x \neq 4 \\ 0, & if\ x = 4 \end{cases}$$ **at x = 4**

Solution:

Checking the right hand and left hand limits for the given function, we have

$$\lim_{x \to 4^-} f(x) = \frac{|x-4|}{2(x-4)} \qquad \begin{bmatrix} \text{for } x < 4, |x-4| = -(x-4) \\ \text{for } x > 4, |x-4| = (x-4) \end{bmatrix}$$

$$= \lim_{h \to 0} \frac{-[4-h-4]}{2[4-h-4]} = \lim_{h \to 0} \frac{h}{-2h} = -\frac{1}{2}$$

$$\lim_{x \to 4^+} f(x) = \frac{|x-4|}{2(x-4)} = \lim_{h \to 0} \frac{[4+h-4]}{2[4+h-4]} = \frac{1}{2}$$

$$\lim_{x \to 4} f(x) = 0$$

$$\therefore \quad \lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x) \neq \lim_{x \to 4} f(x)$$

Thus, f(x) is discontinuous at x = 4.